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REVIEW OF MATHEMATICAL LEARNING TO PROMOTE ALGEBRA UNDERSTANDING AMONG STUDENTS - A STUDY

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ABSTRACT

In contemporary education, **mathematics learning** is the practice of teaching and learning mathematics, along with the associated scholarly research. Researchers in mathematics education are primarily concerned with the tools, methods and approaches that facilitate practice or the study of practice, however mathematics education research, known on the continent of Europe as the didactics or pedagogy of mathematics, has developed into an extensive field of study, with its own concepts, theories, methods, national and international organisations, conferences and literature.

INTRODUCTION

This sounds obvious, but it can cause some difficulties, particularly for definitions with complicated logical structure (like the definition of the limit of a function at a point in its domain). Definitions are not a good place to practice your speed reading. In general there are no wasted words or extraneous symbols in established definitions and the easily overlooked small words like *and, or, if ... then, for all,* and *there is* are your clues to the logical structure of the definition. First determine what general class of things is being talked about: the definition of a polynomial describes a particular kind of algebraic expression; the definition of a continuous function specifies a kind of function; the definition of a basis for a vector space specifies a kind of set of vectors.

THE MEANINGS OF MATHEMATICS

Mathematics is used to communicate information about a wide range of different subjects. Here are three broad categories:

As English mathematician R.L.E. Schwarzenberger says:-

> ***My own attitude, which I share with many of my colleagues, is simply that mathematics is a language. Like English, or Latin, or Chinese, there are certain concepts for which mathematics is particularly well suited: it would be as foolish to attempt to write a love poem in the language of mathematics as to prove the Fundamental Theorem of Algebra using the English language***.

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MATHEMATICS DESCRIBES MATHEMATICS

Mathematics can be used reflexively to describe itself—this is an area of mathematics called metamathematics. Mathematics can communicate a range of meanings that is as wide as (although different from) that of a natural language. Most definitions have standard examples that go with them. While these are useful, they may lead you to expect that all examples look like the standard example. To understand a definition you should make up your own examples: find three examples that do satisfy the definition but which are as different as possible from each other; find two examples of items in the general class described by the definition which do not satisfy it. Prove that your five examples do what you think they do---such proofs are usually short, follow the structure of the definition quite closely, and help immensely in understanding the definition. These examples should be neatly written up so that you can refer to them later. Your own examples will have more meaning for you than mine or the book's when it comes time to review. Occasionally definitions are useful in and of themselves, but usually we need to relate them to each other and to general problems before they can be made to work for us. This is the role of theory.

The relative importance and the intended use of statements which are then proved is hinted at by the names they are given. Theorems are usually important results which show how to make concepts solve problems or give major insights into the workings of the subject. They often have involved and deep proofs. Propositions give smaller results, often relating different definitions to each other or giving alternate forms of the definition. Proofs of propositions are usually less complex than the proofs of theorems. Lemmas are technical results used in the proofs of theorems. Often it is found that the same trick is used several times in one proof or in the proof of several theorems. When this happens the trick is isolated in a lemma so that its proof will not have to be repeated every time it is used. This often makes the proofs of theorems shorter and, one hopes, more lucid. Corollaries are immediate consequences of theorems either giving special cases or highlighting the interest and importance of the theorem. If the author or instructor has been careful (not all authors and instructors are) with the use of these labels, they will help you figure out what is important in the subject.

The steps to understanding and mastering a theorem follow the same lines as the steps to understanding a definition. Mathematics is not a collection of miscellaneous techniques but rather a way of thinking---a unified subject. Part of the task of studying mathematics is getting the various definitions and theorems properly related to each other. This is particularly important at the end of a course, but it will help you make sense of the content and organization of a subject if you keep the overall organization in mind as you go along. There are two techniques I know of which help with this process: working backwards and definition-theorem outlines.

For the mathematics major this question is easy to answer---a large portion of mathematics consists of proofs. The mathematician enjoys the logical puzzle which must be solved to find a proof and obtains aesthetic satisfaction from elegance in proofs. The student who wants to major in mathematics should do so because of ability in deciphering and producing proofs and enjoyment derived from proof well done. The major should also have skill in solving problems and finding applications as well.

But many of you will say ``I'm not a math major; I want applications so that I can use tools from mathematics in my field" or ``I'm just taking this course because it's a requirement in my major and I sort of liked math in high school." Why should you learn about proof?

The applications you meet in other fields are not likely to look exactly like the math textbook applications, which are chosen for their appeal to a traditional audience (largely engineers) and for their representative character. Other applications work similarly, though not exactly the same way. This means that you need to learn how to apply the concepts in your math courses to situations not discussed in those courses. (There is no way that a course could discuss every possible known application: about 500 papers appear every two weeks with applications, and those are just the applications published in the ``mathematical" literature!) To do so you need the best possible understanding of the mathematics you want to apply. Certainly this means that you need to know the hypotheses of theorems so that you don't apply them where they won't work. It is helpful to know the proof so that you can see how to circumvent the failed hypothesis if necessary. One of the major pitfalls of applied mathematics, particularly as practiced by non mathematicians, is the danger of conveniently overlooking the assumptions of a mathematical model. (Mathematicians trying to do applied mathematics are more likely to fall into the trap of making models which have no relationship to reality.)

Many applications consist of recognizing the definition of a mathematical concept phrased in the terms of another discipline---the more familiar you are with the definition, the more likely you are to be able to recognize the disguised version elsewhere. The nuances of definitions are made most clear in the proofs of propositions relating definitions and pointing out unexpected equivalent variants, some of which may look more like a situation in another discipline than the precise form used in your math class.

ALGEBRA

Historically, algebra is the study of solutions of one or several algebraic equations, involving the polynomial functions of one or several variables. The case where all the polynomials have degree one (systems of linear equations) leads to linear algebra. The case of a single equation, in which one studies the roots of one polynomial, leads to field theory and to the so-called Galois theory. The general case of several equations of high degree leads to algebraic geometry, so named because the sets of solutions of such systems are often studied by geometric methods.

Modern algebraists have increasingly abstracted and axiomatized the structures and patterns of argument encountered not only in the theory of equations, but in mathematics generally. Examples of these structures include groups (first witnessed in relation to symmetry properties of the roots of a polynomial and now ubiquitous throughout mathematics), rings (of which the integers, or whole numbers, constitute a basic example), and fields (of which the rational, real, and complex numbers are examples). Some of the concepts of modern algebra have found their way into elementary mathematics education in the so-called new mathematics.

Some important abstractions recently introduced in algebra are the notions of category and functor, which grew out of so-called homological algebra. Arithmetic and number theory, which are concerned with special properties of the integers—e.g., unique factorization, primes, equations with integer coefficients (Diophantine equations), and congruences—are also a part of algebra. Analytic number theory, however, also applies the nonalgebraic methods of analysis to such problems.

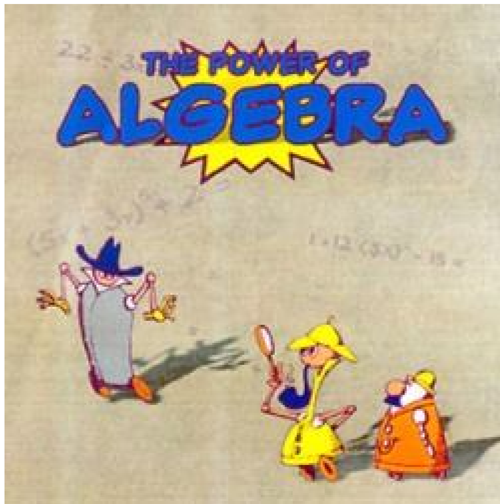
The essential ingredient of analysis is the use of infinite processes, involving passage to a limit. For example, the area of a circle may be computed as the limiting value of the areas of inscribed regular polygons as the number of sides of the polygons increases indefinitely. The basic branch of analysis is the calculus. The general problem of measuring lengths, areas, volumes, and other quantities as limits by means of approximating polygonal figures leads to the integral calculus. The differential calculus arises similarly from the problem of finding the tangent line to a curve at a point. Other branches of analysis result from the application of the concepts and methods of the calculus to various mathematical entities. For example, vector analysis is the calculus of functions whose variables are vectors. Here various types of derivatives and integrals may be introduced. They lead, among other things, to the theory of differential and integral equations, in which the unknowns are functions rather than numbers, as in algebraic equations. Differential equations are often the most natural way in which to express the laws governing the behavior of various physical systems. Calculus is one of the most powerful and supple tools of mathematics. Its applications, both in pure mathematics and in virtually every scientific domain, are manifold.

GEOMETRY

The shape, size, and other properties of figures and the nature of space are in the province of geometry. Euclidean geometry is concerned with the axiomatic study of polygons, conic sections, spheres, polyhedra, and related geometric objects in two and three dimensions—in particular, with the relations of congruence and of similarity between such objects. The unsuccessful attempt to prove the "parallel postulate" from the other axioms of Euclid led in the 19th cent. to the discovery of two different types of non-Euclidean geometry. The 20th cent. has seen an enormous development of topology, which is the study of very general geometric objects, called topological spaces, with respect to relations that are much weaker than congruence and similarity. Other branches of geometry include algebraic geometry and differential geometry, in which the methods of analysis are brought to bear on geometric problems. These fields are now in a vigorous state of development.

APPLIED MATHEMATICS

The term applied mathematics loosely designates a wide range of studies with significant current use in the empirical sciences. It includes numerical methods and computer science, which seeks concrete solutions, sometimes approximate, to explicit mathematical problems (e.g., differential equations, large systems of linear equations). It has a major use in technology for modeling and simulation. For example, the huge wind tunnels, formerly used to test expensive prototypes of airplanes, have all but disappeared. The entire design and testing process is now largely carried out by computer simulation, using mathematically tailored software. It also includes mathematical physics, which now strongly interacts with all of the central areas of mathematics. In addition, probability theory and mathematical statistics are often considered parts of applied mathematics. The distinction between pure and applied mathematics is now becoming less significant.



ALGEBRA AS AN IMPORTANT BRANCH OF MATHEMATICS:-

Algebra is the branch of mathematics concerning the study of the rules of operations and relations, and the constructions and concepts arising from them, including terms, polynomials,equations and algebraic structures. Together with geometry, analysis, topology, combinatorics, and number theory, algebra is one of the main branches of pure mathematics. The part of algebra called elementary algebra is often part of the curriculum in secondary education and introduces the concept of variables representing numbers. Statements based on these variables are manipulated using the rules of operations that apply to numbers, such as addition. This can be done for a variety of reasons, including equation solving. Algebra is much broader than elementary algebra and studies what happens when different rules of operations are used and when operations are devised for things other than numbers. Addition and multiplication can be generalized and their precise definitions lead to structures such as groups, rings and fields.

WHAT IS THE PHILOSOPHY OF MATHEMATICS LEARNING

This question (what is the philosophy of mathematics education?) provokes a number of reactions, even before one tries to answer it. Is it a philosophy of mathematics education, or is it the philosophy of mathematics education? Use of the preposition 'a' suggests that what is being offered is one of several such perspectives, practices or areas of study. Use of the definite article 'the' suggests to some the arrogation of definitiveness to the account given.[1] In other words, it is the dominant or otherwise unique account of philosophy of mathematics education. However, an alternative reading is that 'the' refers to a definite area of enquiry, a specific domain, within which one account is offered. So the philosophy of mathematics education need not be a dominant interpretation so much as an area of study, an area of investigation, and hence something with this title can be an exploratory assay into this area. This is what I intend here.

Moving beyond the first word, there is the more substantive question of the reference of the term 'philosophy of mathematics education'. There is a narrow sense that can be applied in interpreting the words 'philosophy' and 'mathematics education'. The philosophy of some area or activity can be understood as its aims or rationale. Mathematics education understood in its simplest and most concrete sense concerns the activity or practice of teaching mathematics. So the narrowest sense of 'philosophy of mathematics education' concerns the aims or rationale behind the practice of teaching mathematics. 'What is the purpose of teaching and learning mathematics?' is an important question. I have added learning to it because learning is inseparable from teaching. Although they can be conceived of separately, in practice a teacher presupposes one or more learners. Only in pathological situations can one have teaching without learning, although of course the converse does not hold. Informal learning is often self directed and takes place without explicit teaching.

Returning to the question of the aims of teaching mathematics it is important to note that aims, goals, purposes, rationales, etc, do not exist in a vacuum. They belong to people, whether individuals or social groups. Indeed since the teaching of mathematics is a widespread and highly organised social activity, and even allowing for the possibility of divergent multiple aims and goals among different persons, ultimately these aims, goals, purposes, rationales, and so on, need to be related to social groups and society in general. Aims are expressions of values, and thus the educational and social values of society or some part of it are implicated in this enquiry. In addition, the aims discussed so far are for the teaching of mathematics, so the aims and values centrally concern mathematics and its role and purposes in education and society.

However there is a missing element from philosophy of mathematics education that a broader interpretation brings into play, namely that of philosophy. The metonymic structure of the term 'philosophy of mathematics education' brings 'philosophy' and 'mathematics' together, foregrounding the philosophy of mathematics. The philosophy of mathematics is undoubtedly an important aspect of philosophy of mathematics education, especially in the way that the philosophy of mathematics impacts on mathematics education. This is part of the missing element.

In his essay on the subject of the philosophy of mathematics education Stephen Brown (1995) asks a very pertinent question by posing a trichotomy. Is the philosophical focus or dimension:

1. Philosophy applied to or of mathematics education?

2. Philosophy of mathematics applied to mathematics education or to education in general?

3. Philosophy of education applied to mathematics education?

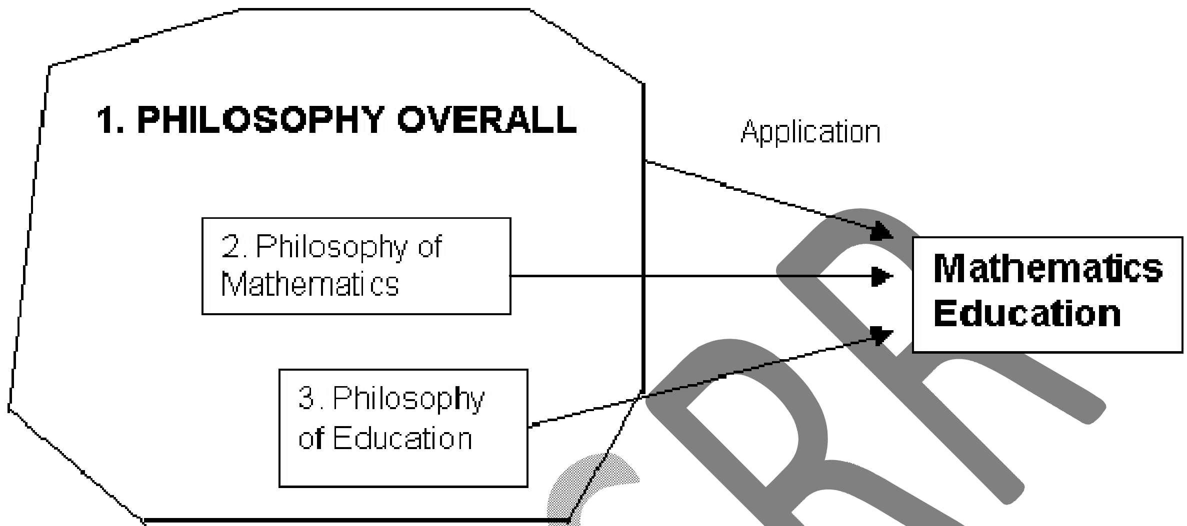


FIGURE 1: DIFFERENT APPLICATIONS OF PHILOSOPHY TO MATHEMATICS EDUCATION

Each of these three possible 'applications' of philosophy to mathematics education represents a different focus, and might very well foreground different issues and problems. However, Figure 1, of course, raises more questions than it answers. It illustrates that applications can be made either of philosophy or of two special branches of it. However what is such an application? The diagram might be taken to suggest that there are substantive bodies of knowledge and applicational activities connecting them, whereas philosophy, mathematics education and other domains of knowledge encompass processes of enquiry and practice, personal knowledge, and as well as published knowledge representations. They are not simply substantial entities in themselves, but complex relationships and interactions between persons, society, social structures, knowledge representations and communicative (and other) practices. .

At the very least, this suggests that the philosophy of mathematics education should not only attend to the philosophy of mathematics. Stephen Brown (1995) suggests that it should also look to the philosophy of Schwab's other commonplaces of teaching: the learner, the teacher, and the milieu or society. So we also have the philosophy of learning (mathematics), the philosophy of teaching (mathematics) and the philosophy of the milieu or society (with respect to mathematics and mathematics education) as further elements to consider.

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